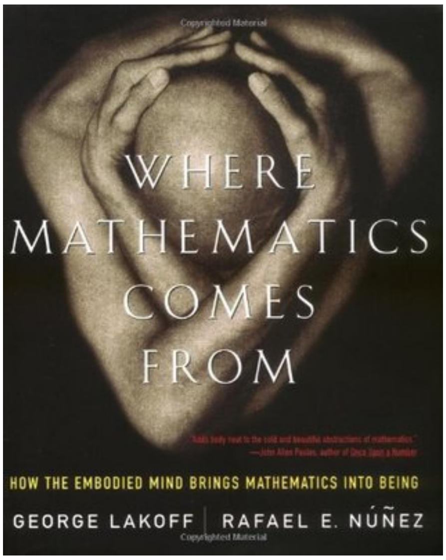

Where Mathematics Come From: How the Embodied Mind Brings Mathematics into Being

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This book is about mathematical ideas, about what mathematics means-and why. Abstract ideas, for the most part, arise via conceptual metaphor-metaphorical ideas projecting from the way we function in the everyday physical world. Where Mathematics Comes From argues that conceptual metaphor plays a central role in mathematical ideas within the cognitive unconscious-from arithmetic and algebra to sets and logic to infinity in all of its forms.

Where Mathematics Come From: How the Embodied Mind Brings Mathematics into Being Details

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From Reader Review Where Mathematics Come From: How the Embodied Mind Brings Mathematics into Being for online ebook

Hamed Zakerzadeh says

It is the worst rating I have ever given to a book! Simply speaking, the whole book is trying to convince you that it has a more realistic explanation of the nature of mathematics, and believe me, it cannot even fake it! The most obvious examples are infinite series and Taylor expansion.

In former, the authors propose a (loosely defined) "metaphor" to show how the infinite series work, which cannot even show the convergence or divergence of the series! It actually gets help from math and is faking to claim so, as if it is its direct conclusion!

For the latter, the Taylor expansion, the authors try to show "in terms of cognitive science" why does the Taylor expansion in one point can construct the whole function everywhere! Everyone with sufficient math knowledge, can confirm that is it not true for most functions, but the authors obviously did not know this fact and got really excited about this invalid claim!

To be positive, the whole long book is a good example of scientific misconduct; one can be more harsh and say that it is a scientific fraud: to claim that you are saying where math comes from, and all you say, is to feed your weak or even wrong arguments with math again.

Carrie says

I find some of the arguments in this book tautological, thought it is difficult to articulate why. The section on an Embodied Philosophy of Mathematics is one of the most interesting in the book. The authors argue against "The Romance of Mathematics" (Platonism, plus some cultural effects) and against Postmodernism as a philosophy of mathematics. Their solution to "what is mathematics" lies somewhere in the middle: every human has certain basic cognitive capabilities. Based on these capabilities, we create metaphors; from these metaphors we build up mathematics. Therefore math is not totally arbitrary, because it is based on universal human neurological characteristics; neither is it universal or somewhere "out there", because it is created and practiced by humans. This philosophy allows room for both cultural and historical contingency and universality.

The writing is very clear, but readers with little or no recollection of calculus might find parts of it difficult.

Bracken says

This says naughty things about George Boole.

10/10

Ushan says

Like language, simple arithmetic has an instinctual basis. People can immediately see whether there is one object in front of them, two, or three, and expect that one object and one more object make two. What of more complicated mathematics? The authors argue that it is built up from simple arithmetic using metaphors, the mechanism that, as Lakoff has argued in another book, is central for cognition. For example, the conceptual jump from real to complex numbers is akin to the conceptual jump in such expressions as "time is money": time is not literally money, but there is a manner of looking at time that shows it being akin to money in some ways. Likewise, there is a manner of looking at pairs of real numbers that shows them as the real and the imaginary parts of a single complex number, akin to real numbers in some ways. Axioms are like essences in folk categories. A car usually has four wheels; there have been some three-wheel vehicles that are like cars; the definition of a car may have to change to accommodate them but not the three-wheelers that are like two-wheel motorcycles. The search is on for a definition that calls what is usually thought to be a car a car, and doesn't call what isn't. Likewise, there have been several attempts to axiomatize set theory; each either included pathological mathematical objects or excluded useful ones. The authors build up a hierarchy of mathematical objects, going all the way to nonstandard analysis, transfinite ordinals and space-filling curves, and describe the metaphors taken at each step. They take Euler's equation $e\pi i=-1$ and explain the metaphorical sense in which it is true.

Frankly, I do not see the purpose of this exercise. Terence Tao proved that arithmetic progressions of prime numbers can be arbitrarily long. This proof is stored in Professor Tao's brain and in the brains of the mathematicians who have read the published proof. We do not know how it is encoded there. Although the particular encoding is specific to human biology, we do not know whether the proof itself is. Right now humans are the only beings on Earth who could understand it; yet it is very possible that within 100 years electronic computers will too. There has been an attempt to construct an automatic discoverer of mathematical concepts; although it did not progress very far, it did discover a great deal more than the overwhelming majority of students of mathematics. If mathematics is defined as the metaphorical extension of instinctual behaviors, then what it discovered is not mathematics, since a machine has no instincts. The authors mention Eugene Wigner asking why mathematics is unreasonably effective in describing the physical world and answer that it is because mathematics exists only in the brains of the physicists who are doing the describing. Yet other disciplines exist in human brains that are far less effective in describing the physical world, such as religion and philosophy; why is mathematics different?

What is also lacking from the book is a sense of how unnatural mathematics is for people. Grade school arithmetic is useless except as a foundation for high school mathematics, which is useless except as a foundation for undergraduate mathematics, which is useless except as a foundation for graduate mathematics. American universities graduate about 1,200 Ph.D.s in mathematics a year, of whom half are U.S. citizens. Even if 10 times as many people are capable of mastering the program but do not, choosing other careers instead, it is still a tiny fraction of the 3-4 million babies born in the United States each year. That's not much for something that supposedly has an instinctual basis, is it?

Carlos says

Reading this book seemed like watching a picture come in and out of focus constantly. The authors start the book with the great promise to explicitly present the underlying metaphors of all of mathematics and they begin quite well. They explain arithmetic from innate counting abilities in humans and clarifying the metaphors by which those innate abilities are extended to all of what we know as arithmetic. Unfortunately, the book starts to see-saw on the following chapter on algebra. The authors start talking about everything in terms of sets and while their explanations are lucid every once in a while, the reader – at least the reader not thoroughly familiar with such concepts as hypersets, hyperreal numbers, quaternions, etc. – start to find less and less room to grasp the concepts that the authors are trying to "clarify". The picture comes back into focus on the chapter of the philosophy of embodied mathematics, but unfortunately the authors try to present the previously incomprehensible examples as evidence of their philosophy. In all honesty the best part of this book was the case study of Euler's equation at the end. The only people I would recommend this to would be mathematicians, as I suspect they would have the necessary background to understand the authors' intent.

Blaine says

Recently completed reading this challenging journey through Lakoff's embodied mind theory with our Philosophy of Math study group at Saint Martin's University. The group, made up of math, philosophy, and computer science professors, struggled with Lakoff's approach to how fundamentals of number, arithmetic, algebra, and infinitesimals are grounded in bodily metaphor and permutations of such metaphors through conceptual blending (for a more detailed look at conceptual blending see Fouconnier/Turner's The Way We Think: Conceptual Blending and The Mind's Hidden Complexities). There was considerable consternation with Lakoff/Nunez in that they often didn't go far enough with explanation to back up their claims and that, according to the math folks, their presentation of math concepts was sometimes plain inaccurate. Some of the group participants were clearly disturbed by the authors' full-frontal attack on Platonic mathematics and disembodied thought and chose not to attend while we read this book. Clearly many are not ready to entertain a worldview where embodiment, immanence, and emergent properties become the basis for the sacred cows of reason, math, and philosophy.

Much of the trouble my colleagues had with understanding and appreciating Lakoff/Nunez's embodied mind approach was due to their lack of background with cognitive science, the philosophy of mind, consciousness studies, and of course systems and complexity theory. Their struggle highlighted to me the truism that, in today's world, specialization has a particular cost to it in that you miss out on the insights gained from contemporary approaches that are multidisciplinary or interdisciplinary such as cognitive science and systems theory. As we know from postmodern thought, we live in a pluralistic world of multiple interdependent-interacting perspectives and bodies of knowledge and to isolate oneself within a specialty or field cuts one off from the often profound insights gained from the cross-fertilization multidisciplinarity offers. My recommendation is to read other introductory books on cognitive science and embodied mind theory such as Howard Gardner's The Mind's New Science: A History Of The Cognitive Revolution or Lawrence Shapiro's Embodied Cognition to get a general sense of the context in which Lakoff and Nunez are doing their work. It also helps to read Lakoff/Johnson's original book on embodied metaphor Metaphors We Live By to get a sense of his approach.

Sean says

A really really excellent book. Okay, yes, I'm weird, I've been interested off and on in philosophy of mathematics for probably almost a decade and a half. Partly to inject some “soul” into the lonely, mechanical, and austere world I had to inhabit as a software engineer, and partly because it’s just irresistible to me the way that the prevailing intuitions and beliefs that we have as to what math is, what numbers are, etc - are so superstitious and absurdly wrong, yet so difficult to explain away, by virtue of being so “deep” (until you have “broken free,” let’s say). It is very much like “God” in that way, although I got bored of God a long time ago.

Anyway, as I understand it, this book inaugurated the “cognitivist” school of thought in philosophy of mathematics, which holds that mathematical systems are constructs grounded ultimately in our cognitive experience and intuition as embodied organisms. A notable implication of this (or motive to take this position) is that there is no transcendental or absolute truth in mathematics. The book brings together perspectives, concepts, and evidence from cognitive science, neuroscience, and linguistics to flesh out its case.

The principal opposing view is known as Platonic realism, and holds that mathematical objects such as numbers, geometrical objects, classes, sets, and whatnot, are ontologically independent of our conceptions, that is, they somehow “really exist,” in some kind of transcendental realm. That we discover them, not invent them, and that mathematical theorems constitute absolute truth. Although this belief was originally closely related to a lot of Plato's other ideas, it has detached itself to a large extent and is actually still to this day a prevailing position among mathematicians, which drives me crazy.

It’s not just philosophically naïve, it (like “god”) also supports a priestly class of elites via the mystique it surrounds mathematics with. It reinforces math’s inaccessibility, alienates would-be students and aficionados, and contributes to the stratification of society into “those who can function in an increasingly technical economy and those who cannot.” I hope math education is being revolutionized by this book as we speak. A good case in point is “imaginary numbers.” We’re taught how to use them at a young age, but if we ask what they are, the teachers don’t know, and tell us to just accept the whole idea on some kind of faith. And if later we get to the point where we learn Euler’s identity, that e to the pi times i equals -1, we’re prepared to accept that it encapsulates some profound truth uniting all truths, so transcendentally mysterious that maybe not even mathematicians can understand, they are just the vessels that transmit it to us in a hollowed-out form - a formula, a sort of scripture to memorize and blindly accept.

Well this book culminates in doing a really stellar job of explaining the conceptual network surrounding Euler’s identity, chosen because it represents the fusion of a wealth of conceptual schemes and unifies several basic branches of mathematics. You might get the impression that the bizarre numerical values of e and pi - far from embodying the nature of the universe or whatever - are simply the place where the dirty side effects of making that fusion as elegant as possible have gotten swept under the rug. There’s always something you have to sweep under the rug somewhere. Knowing we have the option - or rather, that maybe we have no OTHER option - of managing imperfection like that, perhaps instead of despairing of ever understanding the stuff we’re expected to perform robotically, maybe we’d feel empowered to imagine that we can not just perform math, but invent it - that ordinary people can do that, not just geniuses, or at least that maybe we can understand it at a high level even if we don’t master every technicality or get Ph.D.s in it.

In addition to Platonic realism, a secondary target - which I’d been relatively partial to (it’d make sense for it to be a more popular view in computer science) - is formalism, which takes mathematics to consist only in meaningless symbols along with rules for mechanically transforming them in a manner that preserves consistency with a set of expressions that are stipulated to be “true” (where “true” is just another meaningless property that an expression can have). Though true of modern mathematics to some extent, this view doesn’t seem to address so well WHY we construct and use these systems the way we do, which is the main inadequacy that cognitivism sees in it.

The irony of cognitivism is that our tendency to believe in Platonic realism is itself a prediction of cognitivist theory. We along with infants and various other animal species have the neurological ability (in the inferior parietal cortex) to match distinct areas in our visual field to a pattern such that they can appear as repeating instances of that pattern, and we can discriminate between there being more or fewer of those instances by purely visual processing, without “counting,” without numbers. This is called “subitizing.” We can do this up to about four objects at a time, and then we generally need to start employing increasingly abstract conceptual metaphors in order to extend our quantitative abilities, that is, to count, add, etc. These metaphors are ultimately grounded in basic experiences such as putting objects in collections and containers and performing or observing motions and repetitive processes. Number words start out as adjectives but a core metaphor arises that numbers are things in themselves.

Once this happens, numbers need to have more of the properties that objects have. For example if we put two collections together, we get another collection. So, operations on numbers should produce more numbers. This is the cognitive basis for the mathematical property of closure, but closure over most arithmetic operations wasn't achievable until the number system was extended to include zero, negatives, rationals, irrationals, and imaginaries and to form infinite sets. And you still can't divide by zero! Rather than encapsulating pure elegant truth, mathematics finds itself continually imperfect in ways that fuel the need for new inventions to, let's say, patch its bugs. And we keep reifying our metaphors.

The most ridiculous example may be the set of hyperreal numbers. The regular real numbers have no gaps, right? They're continuous, with infinite precision at every scale and magnitude, right? Yes, of course. And, also no, the real line is actually pretty much entirely gaps. Each real number ‘needs’ to have a ‘monad’ around it consisting of let's just whimsically call it “infinity factorial bazillion universes full of infinities worth” of ‘infinitesimals,’ which are numbers whose values are closer to zero than that of any real number except zero, even though there’s already an infinite number of real numbers that are already infinitely close to zero. And none of these monads can overlap with their neighbors. You can get away with just one level of infinity's worth of infinitesimals, like in Leibniz' version of calculus, but you have to add an arbitrary axiom to the number system just to keep the rest away. These infinitesimals, plus a bazillion sets of infinite numbers (because the infinitesimals need inverses, for closure, of course) combine with the real numbers to form the hyperreals. They’re very carefully and weirdly formulated to manage that contradiction about the reals having gaps - to make it a conceptual (that is, an “apparent”) contradiction instead of a formal one. But why is this insanity needed? To fix an obscure fatal issue in which calculus and mathematical logic were discovered to violate each other. But once that bug is fixed, you can't do calculus anymore, at least not with perfect precision, because all those infinitesimal terms accumulate into a massively unwieldy mess and jam the gears of your algebra. They don't teach the hyperreals because they're kind of a useless embarrassing nuisance.

A lot of the book revolves around the ubiquity of infinity in math, from limits in Newtonian calculus to Cantor's transfinite numbers for describing different "sizes" of infinite sets. Maybe you're like me and have puzzled hopelessly about what to make of infinity only to make yourself physically queasy. Maybe you feel like you can sort of accept the idea of "potential infinity," that there's something not intolerably ungrounded or defiant of intuition about that, but that surely "actual infinity" either cannot be possible or must be truly transcendental or divine, beyond intellectual grasp. Actually, "actual infinity" works a lot better in our loose intuitions than in the rigor of mathematics (not to mention external reality), and much of that rigor consists of elaborate convolutions to try to apply our intuitive metaphor of infinity to its unforgiving structures. Infinity is our own invention, it's just a metaphor, and when this book points out the quotidian ways you use it, it'll make perfect sense. You'll probably be disappointed but your gastrointestinal tract will thank you.

It's such a weird mix of humility and hubris the way humans are hypnotized by our own conceptions. Ha, that's a lot of H words. H by the way is the first hyperreal number that's bigger than all real numbers (even though they are infinitely big already). (Whatever that means). H can be raised to infinite powers of itself. If numbers "really exist" then Hopefully their transcendent realm is big enough for all that extravagant excess.

The strength of the book is its analytical clarity, and for such a dry and somewhat technical subject matter, the writing is so amazingly lucid that it's a surprising pleasure to read. Conceptually, they've thought through some ideas that are very difficult to express, and they express them with that kind of mastery that makes them seem easy and obvious. A relative weakness is one they acknowledge from the start: that it was a new field of study and much more evidence needed to be collected. In fact they don't directly cite much evidence at all, although they do make extensive use of a basic set of concepts about cognition that seem well established. It's not always easy to know when they've developed their own ideas and when they are tying into other research. The argument is more analytical than empirical, and mentions neurological and linguistic considerations a lot less than I expected.

Also, if you were inclined to be skeptical from a Platonic realist's point of view, it's probably easy to claim that these schemas describing our cognition - to the extent they form a well-definable mapping onto mathematical structures - are themselves mathematical, and these metaphors are thus themselves mere instantiations of 'a priori' order and meaning. Perhaps there's a deep relation with the structure of reality to why we can develop rigorous formal ideas based on object collection more easily than on, like, tasting fruit. The authors claim it's surprisingly difficult to identify grounding metaphors, since many embodied experiences like describing apples don't lend themselves to inventing arithmetic too well. I'm skeptical that it was really that hard to backtrack from counting and division to apportioning meat to your clan or whatever.

But if you really step back you can note that there's really no such thing as objects at all except as conscious artifacts of our neural processing as organisms in an environment. So in some sense there's nothing more to say about numbers once you've grounded them in such objects. Maybe other things go deeper, but numbers don't. The book takes its ability to say so much so clearly by operating at the human level instead of the metaphysical one: literally, where does math come from, not where does order and structure itself come from. It must've been anything but obvious how to delineate this boundary! But doing so is maybe the real crux. It's one thing to feel like that begs some deeper questions way down the philosophical rabbit hole, and quite another to try to swallow up more of the subject matter into a turgid metaphysical abyss than is necessary, under the misguided notion that that kind of oblivion is, like, a signature property of "truth."

I'm content to believe there can be mystery without truth, and that it's better that way.

Jared Leonard says

Very educational even if you don't have a strong background in math. If you are a fan of Lakoff's previous works (like "Metaphors We Live By" or "Philosophy in the Flesh") this is definitely a must read. The primary argument is that traditional conceptions of mathematics are incorrect because they make the fundamental mistake of presuming that axioms "just are" and have no subjective context. The authors do a great job of laying down the groundwork for showing that the mind has an innate ability process very small mathematical quantities and basic operations of addition and subtraction. They outline the conceptual metaphors which are formulated by these innate abilities and show that all of mathematics can be extrapolated from them.

Alex Lee says

Cognitive linguistics has at its underlying aesthetic the very literal understanding that how we think of things is what they are. This follows post-structural rhetoricians like Paul Ricoeur who argue that the connective tissue of language is metaphor -- where metaphor is the substantiation of the naked copula form *is* through content. We forget the form of the copula in metaphors and thus experience the content as a variation of the copula form instead of being the actual connection. In other words we understand our world through representations, never understanding that an ontologically reified point of view is only possible because metaphors position the copula through its latent content so that the form of the copula becomes seen as the "ding as such". In other words, representations only appear to be representations because one of the formal representations comes to represent nothing but the pure presence of its own linguistic connectivity.

Having said this, I was surprised (but also not surprised) by the comments below. Many people were confused by this book, blaming either the psychologists for not living up to their expectations (of not being neurologists), or blaming the thickness of the mathematical concepts presented. We often think of the pure formalism of math as being objectively isometric (as one reviewer said) to the proposition that reality is always present beneath our representations. One key connection that Lakoff and Nunez said repeatedly is that many mathematical formalisms (such as zero, negative numbers, complex numbers, limits, and so on) were not accepted even long after their calculatory prowess was proven effectual... what made these concepts acceptable wasn't their caculatory significance, but rather their introduction to the cannon of mathematical concepts via metaphoric agency. For instance, we take zero for granted as being "real" even though we understand it to not be a true number. It only was after a new metaphoric concept was presented for zero to be sensible (numbers as containers and origin on a path) was then zero incorporated into the cannon of what was acceptable. This understanding proves to be the very "twist" needed for Lakoff and Nunez to write this book. While many of the concepts are perhaps difficult for some of us non-mathematicians to grasp, I found their presentation to be concise and illuminating. Their tabulatory presentation of metaphors side by side allow us to grasp the mapping of logically independent factors from one domain into another. This basic movement is in fact a methodology they may have picked up from analytic geometry as invented by Rene Descartes: the translation of continuums into discrete points.

While it is understandable that they trace the building of conceptual metaphors via simple to the more complex, I did find their delay of speaking of analytic geometric to be confusing. When a topic is presented I want it to be explained, rather than having to wait half a book to read on it again. This is really my only possible complaint.

Overall, this book helped me connect the observation of formalism being prevalent as an organizing feature of pretty much all procedure and knowledge formation today with the root of that formalization, being the atomization of discrete epistemes of knowledge, whether that knowledge is granular or point or vector, or some other kind of rigor. We can also thus understand mathematics as being synthetic, contrary to what most philosophers in the west (excluding the great Immanuel Kant, Alain Badiou and Gilles Deleuze) understood.

Today, through our rockstar mathematicians and physicists we revisit the old Platonic hat that math is somehow natural, only apparent in our minds and yet more real than anything else this world has to offer. This is a troubling and definitely cold and etymologically naive sentiment. It's mysterious that anything in this world is the way that is, let alone consistent as though following laws, but that isn't any reason to be hypnotized by our own intellectual conceptions. As Lakoff and Nunez point out, while some math is applicable in the physical world most conceptual math remains beyond application of the physical world, as there is no physical correlation with those domains. Such application may be possible in alternate universes, but such universes remain the sole conception of our mind.

In other words, how we think of something is what we understand it to be, that is true, but it's also how we experience what we understand to be to be what it is. To get into that deeper thought requires an unpacking of the most erudite philosophical concept of all -- that of the number One, arguably the only number there ever has been and in fact the only thing there has ever been. Understandably this is beyond the scope of mathematics itself, or at least beyond the tenants of what most mathematicians are willing to go. I don't want to belabor the point here, but I will state that the case study at the back of the book is quite compelling. If Euler's equation may work in formal procedure alone, but as Lakoff and Nunez point out, the construction of that equation is only possible through the discrete projections of layered metaphors to understand equivalence of conception regardless of the different construction domains these metaphors originate from (logarithms vs trigonometry, vs Cartesian rotation vs complex numbers)... ultimately a unity is made possible because such closure is driven by the singular domain of our minds. In our minds, with their ornate metaphors, their clearly trained disciplines and their innate mechanisms of spacial orientation, we are able to combine complex concepts into the most brilliant of abstractions.

As such this book may be too difficult for most of us to read, because it requires we re-orient our thinking along different parameters, different assumptions about who we are and what we are doing when we study and create math. This probably won't jive with most people, as it seems for most people, knowledge is less about reworking what they already know into a new arrangement, and more about filling in gaps in the arrangements they already have.

I'm not saying that this cognitive linguistic approach is equivocally true, I'm saying that truth is more than how we arrange something, but the entire range of what we can conceive of to be a relation that brings to light new connections. In the end, I think for most of us, the only legitimatizer of reason remains one's singular emotions, of what feels to be acceptable. To get around this, requires the most stern of discipline and the most unabashed eagerness to learn something new. This is also a reminder that math is not formal procedure as we learned long division in our elementary grades. Rather, math is the unabashed conceptualization of formal arrangements in their absolute complexity. In this way, even understanding how highly educated mathematicians think of math is illuminating to how you and I can understand something (ourselves and the universe) in new light. That alone is worth reading this book.

So do read this book because it's beautiful, but also read this book because it's another way of considering something you already think you know. After all, learning isn't a matter of facts. Facts are boring; the world is full of facts we can never memorize (such as where your car was on such and such date and time. Kind of useless, except in special cases, such as in the immediate). Learning is the mastery of how to conceptualize, how to arrange information and how to further that arrangement through metaphor of what *is*.

Peter D. McLoughlin says

Although I am a Mathematical Platonist. I couldn't help but be fascinated by Lakoff account of how concrete metaphors from the body and everyday experience inform our mathematical abstractions of the most aetherial and least earthy types. My only answer to Lakoff repudiation of platonism for a cognitive origin of mathematics is where does the regularity of the world (that cognitive patterns are built on) originate. An excellent Book.

Dav says

I consider this book to be essential reading for anyone attempting to seriously understand Mathematics. In fact this book or should probably be required for anyone teaching Mathematics!

I've long believed that there was no way to break down thought into discernible mechanistic-like chunks and analyze the thought process in a non-hand-waving manner. I am delighted to discover I was wrong about this. It turns out cognitive scientists have developed what seems to be a very solid method and vocabulary for doing just that, and this book explores using the methodology to analyze Mathematical ideas. The results are impressive. The authors do a good job in arguing their central notion that Mathematics can be understood as something that emerges not from an objective discovery of universal abstract truths, but from a sometimes messy set of thought processes building upon a few basic neurological capabilities, and these basic neurological capabilities are firmly embedded in our physical reality. Along the way of making this point, they do a thorough job of laying bare the conceptual trickiness in certain Mathematical ideas.

The first third of the book was a little tedious. It felt like programming in Assembly language, the "close to the metal" language of computers: the operations and ideas were extremely simple and repetitive. The payoff for making it through this however was a wonderful jaunt through the land of Infinitesimals and the battles between Geometers and the Discretization program (I think a good companion to this book would be the Donald Coxeter biography King of Infinite Space: Donald Coxeter, the Man Who Saved Geometry). The last third of the book is a case study of all the ideas inherent in understanding Euler's beautiful and somewhat mysterious special identity formula e**(pi*i)+1=0. I'm setting the book aside for now until I have more spare time to commit to consuming this last part.

I come away from all this with the wish that there were a Wikipedia-like reference that gives every Mathematical idea this level of analysis. If you can speak this language of the cognitive scientist, it seems like the most clear way to express any Mathematical idea.

Jeff says

An utter disappointment but not devoid of value. I doubt i'll bother to continue reading, so here are my initial thoughts.

I'm far too dim to understand Lakoff + Núñez's ideas.
Or maybe they're not saying anything other than people have to use language to express and explain mathematical ideas and language is entirely metaphorical.
Or maybe they're saying that mathematics is entirely metaphorical and language is fundamental.
There's a philosophical chicken-and-egg problem with this entire book.
Or maybe i don't know what i'm talking about.
Or maybe i don't know what L+N are talking about.
I'm a 25 watt bulb in an array of Klieg lights.

I've always wanted to share my love for this crazy Chick Publication and now might be my best/only chance. While reading L+N's first few "mappings" of a "metaphor" from a "source domain" onto a "target domain," i couldn't stop thinking of the 4th-from-last page of Are Roman Catholics Christians? (note: Chick Pubs are only good if you wanna laugh at a fundamentally disabled mind]

Obviously L+N's book is yet another that i'm unqualified to critique so i searched the interwebs for reviews and found one by a professional mathematician that i wish i'd written.

Assuming you hold no strong prejudices regarding "cognitive science" or "the embodied mind" or "Platonism" or "Aristotelianism," maybe the following quotes from James Madden's article (*Notices of the AMS*. 2001;48[10]:1182–88) will convince you to be on my side of this futile debate.

> Presumably, when an individual is engaged in mathematical work, that person is guided by metaphors that are somehow represented in his or her own brain.... Unfortunately, Lakoff and Núñez do not provide any illustrations of what they suppose goes on in "real time," so this is about as much as I can say.
>
> This brings me to my first main criticism concerning the metaphor hypothesis: What is the quality of the evidence for it?... I would like to have seen direct support for the metaphor hypothesis from the observation of mathematical behaviors. (1184-5)

"After a while, the notion of metaphor seems to become a catchall." (1185) Did Madden grow as tired of the vagueness and ubiquity of the term metaphor as i did?

Madden damns with faint praise and cuts deep into the heart of L+N's premise:

> The idea that metaphors play a role in mathematical thinking is quite attractive, but what is needed is a notion specific and precise enough so that people working independently and without consulting one another can discover the same metaphors and agree on the functions they perform. I do not think we have this yet. (1185)

And a summary of sorts:

> If I think about the portrayal of mathematics in the book as a whole, I find myself disappointed by the pale picture the authors have drawn. In the book, people formulate ideas and reason mathematically, realize things, extend ideas, infer, understand, symbolize, calculate, and, most frequently of all, conceptualize. These plain vanilla words scarcely exhaust the kinds of things that go on when people do mathematics. (1187)

Madden does give L+N credit for "shar[ing] with us their intuitions about the way the mathematical mind operates," but he's not convinced that anything they said has been scientifically tested even though it's presented to us as if it had been proven.

Jrobertus says

I read Lakoff's earlier book, Metaphors we live by, and loved it. This is way more in depth. It starts strong with an introduction to what are best thought of as "hard wired" cognitive faculties like "subitizing" (instantaneous number recognition, this is also seen in other animals to a lesser extent). The authurs then build on visual schema to lay the basis for useful metaphors to comprehend higher math, at least through arithmetic and elementary logic. After that, the book goes off the rails. The embodiment idea is obscured and the authors do a poor job of making their increasingly complex case. More examples would be useful to understand the math, but they seem more concerned with showing off than being clear. The parts that were good, make the early part of the book useful so I am scoring it higher than it may deserve.

Joseph says

Lakoff and Nunez are cognitive scientists with a deep interest in mathematics and in this book, they try to explain mathematics from a cognitive perspective. The result is fascinating. I am a mathematical realist and, as such, I have some philosophical disagreements with the authors of this book, but their explanation of the metaphors involved in some mathematical concepts I found fascinating. Furthermore, I think their ideas of embodied mathematics is fully reconcilable with an Aristotelian hylemorphic concept of reality (if not a Platonist concept of reality). Lakoff and Nunez clearly subscribe to materialism and scientism, and they conflate the brain and the mind. However, despite some clear flaws, I think all mathematics teachers and everyone interested in the ideas of mathematics should read this book.

Stan Murai says

George Lakoff, a cognitive linguist, and Rafael E. Núñez, a published their work "Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being" in 2000 as a 'cognitive science' study of how mathematics is 'embodied' and based on 'conceptual methaphors'. Cognitive science in an interdisciplinary approach towards studying how the mind works and the processes which characterize it, namely thinking. In this book, mathematics is regarded as embodied or shaped by aspects of the body. That is, the neurological or perceptual system built into the brain. However, any neural engrams involved in mathematical processing are described only at a high, abstract, hypothetical level. The book is primarily about the conceptual metaphors that underlie the foundations of mathematics. One of the most important of which the authors call the Basic Metaphor of Infinity (BMI), which is used to represent many areas of mathematics that deal of endless, unlimited sequences or processes described in numbers. I found this to be a fascinating, intriguing work, although I imagine some professional mathematicians and philosophers would still regard the substance of mathematics as existing outside of the minds of mathematicians, a view the authors reject as Platonism. George Lakoff, the better known of the two authors, would be recognized by his recent work on how conceptual metaphors can affect the narrative of political discourse. His observations on how conservatives have overtaken and undermined liberals and progressive by framing the political discussion to their advantage; e.g. the day George Bush took office "tax relief" in place of "tax burden" or "tax responsibility" began to frame how the issue would be viewed.
